dimensional deflection parameter was calculated for various plate thickness to lateral dimension ratios. These results are presented in Fig. 3. They show excellent agreement with the Reissner theory. Furthermore, the same example in Fig. 2 tested the limitation of the various mesh sizes. Results of the twisting moment m_{xy} seemed to agree with both the classical theory and Reissner theory as the mesh size decreased.

2) A homogeneous circular plate with built-in edge. Deflection was plotted against the nondimensional parameter x/a in Fig. 4. By comparing the present result with the classical theory,[7] an increase of maximum deflection due to transverse shear of 2.6% for the thickness ratio 0.01 was obtained, which appeared to be in satisfactory agreement with Reissner's theory.

3) A circular sandwich plate with built-in edge. The properties along with the results are shown in Figs. 5 and 6. The deflection w and the stress resultant m_x agree well with the results of the solution given by Ref. 8.

4) Cylindrical bending of a laminated composite plate subjected to a sinusoidal load. In Fig. 7 the coefficient β of the maximum deflection w was computed and compared with the value reported by Pagano,[7] excellent agreement was obtained.

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New Approach to the 3-D Transonic Flow Analysis Using the STAR-100 Computer

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I. Introduction

FINITE difference schemes have been successfully used to solve governing equations for transonic flow. The development of new algorithms in the early- and mid-1970s have lead to useful design tools for both 2-D airfoils[1,2] and 3-D wings.[3,4] However, because of the number of computations required and capacities of the current generation computers, only the 2-D flow analysis codes may be considered as cost effective. There is a great deal to be desired for the 3-D flow analysis codes. Before the supercomputer pursued by NASA Ames becomes operational, the newly matured vector processing machine STAR-100 shows strong potential of offering a wing designer the turn-around time that is so precious and sometimes vital. There has been only one document about solving the transonic flow equations on a vector machine.[5] The findings are somewhat disappointing. A new explicit scheme had to be developed in order to vectorize the computation. The convergence rate is approximately three times slower than the implicit successive line over-relaxation (SLOR).

This Note presents the findings of a Boeing team that investigated the potential of the STAR-100 computer in solving transonic flow problems. A scheme known to many numerical analysts as the 2-cyclic approach was adopted for solving the 3-D transonic small disturbance equation. The convergence rate was found to be no slower than the SLOR. Therefore, the speed advantage of STAR-100 over scalar machines such as CDC 6600 and 7600 can be realized.

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II. Governing Equation and Solution Procedure

The transonic small disturbance equation

$$[(1-M_\infty^2)\phi_x - \tfrac{1}{2}M_\infty^2(\gamma+1)\phi_x^2]_x + [\phi_y]_y + [\phi_z]_z = 0$$

was written in the so-called conservative form. The Cartesian coordinate system was used to solve this equation numerically where y- and z-derivatives are always represented by central differencing. The x-derivatives are represented by either central differencing or backward differencing depending on the sign of $1-M_\infty^2-M_\infty^2(\gamma+1)\phi_x$. Classically, successive column relaxation has been used to solve the problem where only the ϕ values along a column of constant x,y coordinates, say, are solved simultaneously. All the values in the surrounding columns are considered to be known. The flowfield is swept column by column in an orderly fashion until convergence is achieved.

While the successive column relaxation has been used successfully for scalar machines, there are no easily identifiable vector type operations involved. Creating vector operations is first a programming consideration which in turn causes some modification to the numerical algorithm. In this case, the program that employs the SLOR had to be almost totally rewritten, but the modifications to the algorithm were relatively minor. The first attempt treated the whole y-z plane as a single vector, giving for this problem vector lengths of $M\times N$ for the generation of tridiagonal linear equations and M for the solution. M is the number of y meshes and N is the number of z meshes. This formulation altered the algorithm such that only ϕ values of previous iterations were used in the y-differencing as compared with mixed previous and current iteration values being used in SLOR. This resulted in a stability problem that prevented convergence in nonuniform mesh distributions. This difficulty gave rise to the use of the 2-cyclic method.

The 2-cyclic method splits the y-z plane into two sets of columns, the odd-numbered ones and the even-numbered ones. First, the standard column relaxation is carried out on all the odd numbered columns treated as a single vector. This cut the vector lengths to $(M\times N)/2$ for generation of equations and $M/2$ for solution. Velocity potentials obtained from the previous iterations were used to form y-derivatives for the odd-numbered columns. Then, the column relaxation is carried out on all the even-numbered columns treated as a single vector. The new ϕ values just obtained for the odd-numbered columns are used to form the y-differencings. This

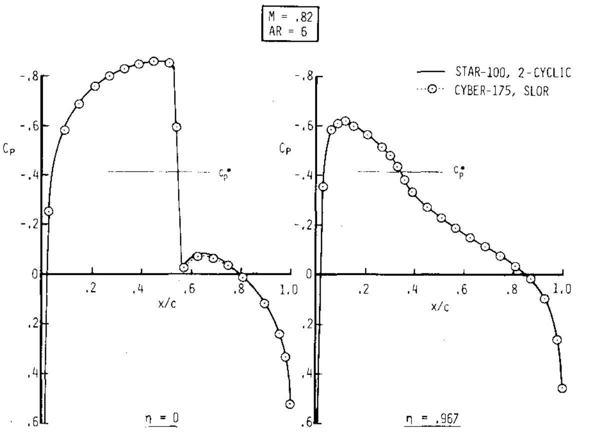

Fig. 1 Pressure distributions on a rectangular wing at $M_\infty = .82$, $\alpha = 0$ deg.

approach proved to be very satisfactory and some preliminary results are presented in the next section.

A short explanation of the tri-diagonal solution in vector mode is in order. Solving a single set of tri-diagonal equations is inherently a recursive operation and thus cannot be vectorized in any reasonable manner. In this case, since all the equations for half the y-z plane were derived simultaneously, one has the prospect of solving them in parallel. In this case, one forms vector operations of length equal to the number of sets of tri-diagonal equations. For the STAR-100 this is a very short vector ($M/2$) but it is considerably better than running in scalar mode.

III. Results

A computer code was developed for the STAR-100. All the basic operations used in the SLOR are retained, including the time consuming process of determining the local flow character, i.e., subsonic, sonic, supersonic, or shock. The code for the 2-cyclic approach has only 5% in common with the first version in which column relaxation was carried out simultaneously for all the columns in an x-plane. This is interesting because their simulations on the scalar machine differ only by one simple DO loop. This experience tells us that numerical research for the vector machine really ought to be simulated on scalar machine first. The well-matured software of scalar machines makes it much easier to change the direction of approach.

The code was used for a rectangular wing of $AR = 6$ with constant NACA 0012 sections.[6] A mesh of $64 \times 28 \times 20$ was used. The pressure distributions at $M_\infty = .82$, $\alpha = 0$ deg is shown in Fig. 1 for the wing root and the wing tip. It may be seen that there were slight differences between the results obtained from the SLOR (CDC CYBER 175) and 2-cyclic approach (CDC STAR-100), especially behind the shock. When the 2-cyclic approach was carried out on the CDC CYBER 175, the same character was observed. It is concluded that the variation of the pressure distribution was a result of the 2-cyclic approach, not that of using the STAR-100 computer. The computer time used for each case is shown in Table 1. The numbers of iterations required to reach the same level of convergence are nearly identical for SLOR and 2-cyclic approach. Hence, the high speed of operation on STAR-100 was not offset by adverse effects from the algorithm change. A speed increase factor of (3.4) was achieved. It is believed that a much better gain may be achieved if longer vectors could be used.

Table 1 Computer time used for SLOR and 2-cyclic technique

	SLOR (CYBER-175)	2-cyclic (STAR-100)
CPU time, s	261.1	76.5
No. of iterations to reach convergence	266	265

IV. Conclusions

From the exercise carried out at Boeing, we discovered that in order to use efficiently a vector machine such as STAR-100, the function of the code should be tested first on a scalar machine. This is mainly because of the mature software for scalar machine and its great flexibility to allow the user to modify program functions. A major rewrite (95% of the code) had to be made when a simple switch from SLOR to 2-cyclic approach became necessary on the STAR-100. The development of an explicit scheme such as the one by Keller and Jameson[5] does not seem to be necessary, especially when the explicit scheme converged three times slower than the implicit SLOR even for a 2-D problem. On the other hand, the short and moderately long vectors ($M = 28$ and $M \times N = 560$) used in this study obviously paid some penalty for start-up time. Further studies of such problems as how to increase the vector length are necessary to fully take advantage of the vector computer such as the STAR-100.

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Conservation Errors in Axisymmetric Finite-Difference Equations

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I. Introduction

THE prediction of many flows of practical interest requires that the solution be carried out numerically. The governing partial-differential equations are replaced by finite-difference equations. The errors that can arise when such a procedure is followed are usually attributed to numerical

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